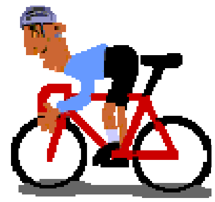


Project: Pedal Power
Teacher Notes: Save this as a key, then delete answers for use with students.
Problem 7 is the most difficult, and perhaps suitable for the most capable students.

Partners: ______________________ and _____________________________

From these exercises you will learn some of the mathematics of bicycles and perhaps find an interesting topic for your presentation.

1. The diameter of the circle made by the front wheel of your bicycle depends upon how much you turn the handlebars.

Angle	Diameter
0°	-
10°	461in.
20°	234in.
30°	160in.
40°	124in.

a. Is the relationship between the angle of the turn and the diameter of the circle a function? Explain why or why not. *(Answer: Yes. The data points pass the vertical line test when graphed.)*

b. What is a reasonable domain for the relationship? Explain. What would be a reasonable range? Why?
(Answer: Domain: (0,40°) Turning 0° wouldn't create a circle and turning much more than 40° would tip over the bicycle.
Range: (124") A 40° turn makes a 124" circle, and as we get closer to a 0° turn, the circle gets larger and larger.)

2. Bicycle racecourses have *banked*, or sloped, curves. The bank keeps the tires perpendicular to the road around a curve to prevent skidding. The approximate speed at which a bicyclist should enter a curve banked at b° is given by the formula below.

$$v = \sqrt{0.171rb}$$

v = the speed in meters per second
r = the radius of the curve in meters
b ° = the measure of the angle at which the curve is banked

a**.** How fast should a bicyclist enter a 12° banked curve with a radius of 40 m?
(Answer: $v = \sqrt{(0.171*40*12)} \approx 9.1$ m/s)

b. Suppose you design a racetrack that has a curve with a radius of 10 m. You want the average speed on this part of the track to be 11 m/s. At what angle should you bank the curve?
(Answer: $11 = \sqrt{(0.171*10*b)} \Rightarrow b \approx 71°$)

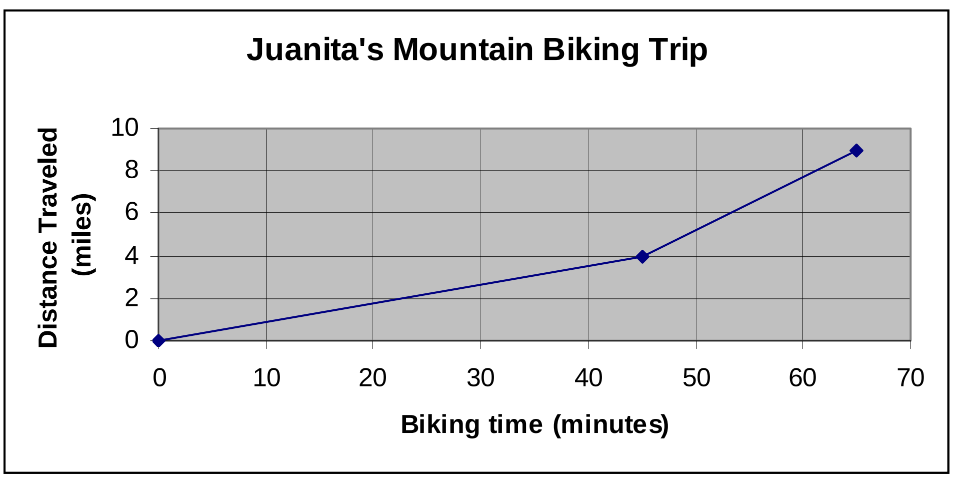
c. You decide to design another curve and bank it at 7°. You want the average speed on this part of the track to be 12 m/s. With what radius should you design the curve? *(Answer: 12 = √(0.171*r*7) ⇒ r ≈ 120 m)*

d. The following is a sample spreadsheet based upon the track-banking formula. If you created a similar spreadsheet as part of your presentation, you could experiment with various radii and banking angles.
Use the velocity information in the spreadsheet to write a proportion to convert a velocity of 20 m/s to miles per hour. Round your answer to the nearest tenth.
(Answer: 14.3 m/s)/(32 mi/hr) = (20 m/s)/v ⇒ v ≈ 44.8 m/s

Banking Bicycle Racecourses $v = \sqrt{0.171rb}$

Radius of Curve (meters)	Angle Banked (degree)	Velocity (meters/sec)	Velocity (mi/hr)
20	10	5.8	13.1
20	20	8.3	18.5
20	30	10.1	22.7
20	40	11.7	26.2
20	50	13.1	29.3
20	60	14.3	32.0

3. During the summer, Juanita likes to mountain bike. The graph shows the distance traveled on a 9-mile loop trail as a function of time.



a. Write a piecewise function for the graph and give the function's domain and range.
(Answer:
f(x) = (4/45) x, if $0 \le x \le 45$
f(x) = (5/20) x – 7.25, if $45 < x \le 65$
Domain: [0,65] Range: [0,9])

b. b. How long does it take Juanita to complete the trail?
How long does it take her to reach the halfway mark of the trail?
(Answer: 65 minutes
From graph: 47-48 minutes or solve y = (5/20)x – 7.25 which $\Rightarrow$ x = 47)

c. Is Juanita's speed constant?
How can you tell by looking at the graph?
What is the average speed for each section of the trail?
What might be the reasons for any change in speed?
(Answer: No. The slope of the distance/time line changes.
First section: (4mi/45min)(60min/hr) $\approx$ 5.3 mi/hr.*
*Second section: (5 mi/20 min) * (60 min/1 hr) = 15 mi/hr.*
May have changed from uphill to downhill, or from rough surface to smooth.)

4. In 1992, only about 20% of adults and 5% of children wore protective helmets while bicycling. (Do you think these numbers have changed since 1992?) In this exercise, you will investigate what happens if you fall.

a. When a person of average height sits on a bicycle, the head is about 5.3 feet above the ground. Suppose such a person falls over, from a bicycle at rest. If *d* is the distance in feet, use the function $d(t) = 16t^2$ to estimate the time in seconds that it takes for the head to hit the ground.
(Answer: 5.3 = 16 t^2 $\Rightarrow$ $t^2 \approx .33$ $\Rightarrow$ $t \approx .58$ sec)

b. The function v(t) = 32t gives the speed in feet per second of an object that falls for t seconds. Estimate the speed at which the head of a person hits the ground.
*(Answer: V(.58) = 32 * (.58sec) $\approx$ 18.56 ft/se)*

c. A cyclist not wearing a helmet will generally have lasting brain damage if the cyclist's head hits the ground at more than 18 ft/s. Does the average person exceed 18 ft/s when falling off a bicycle?
(Answer: Yes)

5. **If there were no friction from the air or the road, a cyclist would coast forever at a constant speed. A cyclist must supply energy to maintain a constant speed. The power (in watts) that a 75 kg cyclist on a 10 kg racing bicycle maintaining a constant speed must supply is** $P = 0.2581v^3 + 3.509v$**, where** ***v*** **is the speed in meters per second. The** $0.2581v^3$ **term represents the power required to overcome air resistance, while the** 3.509v **term represents the power required to overcome the rolling resistance of the tire.**

a. In a two-hour bicycle race, Andy rides at an average speed of 11 m/s (25 mi/h).
How much power is used to overcome air resistance?
How much power is used to overcome rolling resistance?
How much total power must he supply to ride at this speed?
(Answer: 0.2581 8 $(11)^3 \approx 343$ *watts*
*3.509 * (11)* ≈ 38.6 *watts*
about 382 watts)

b. Which do you think is more important to a bicycle racer, rolling resistance or air resistance? Describe two things a racer could do in order to reduce the air resistance.
(Answer: Air resistance, since velocity is cubed, and racers have a high velocity. Crouched position, tight clothing, a fairing, losing weight, etc.)

c. When Andy rides his bike to school, he supplies 0.1 horsepower (74.6 watts) to the bicycle in order to keep it moving at a constant speed. Use a graphing calculator and the horsepower formula above to find his speed.
(Answer: Graph $f(v) = 0.2581v^3 + 3.509v$ *and f(v) = 74.6, and use "Trace" to see where they intersect.* $V \approx 6.4$ *m/s (or 14.3 mi/hr).)*

6. **The average speed** ***v*****, in meters per second, of a rider in a 40 km bicycle race is given by the function** $v(t) = \frac{40000}{t}$**, where** ***t*** **is the total time in seconds that is takes the cyclist to complete the course. Suppose the racecourse is flat and two 75 kg cyclists ride identical bicycles at a constant speed. The power** ***P*** **in watts that each cyclist supplies is given by the function**
$P(v) = 0.2581v^3 + 3.509v$.

Racer	Time
Greg	1 h 12 min
Lance	1 hr 14 min

a. What was Greg's average speed during the race?
(Answer: $V = 40{,}000$ *m/4320 sec* ≈ 9.3 *m/s (20.8 mi/hr)*

b. How much power did Greg supply to the bicycle? (Your answer should fall between 200 and 250 watts.)
(Answer: P(9.3) ≈ 240 watts)

c. Find the composition of functions, *(P o v) (t),* in other words, *P(v(t))*.
Explain what this function represents.
(Answer: P(40,000/t) = 0.2581 (40,000/t)³ + 3.509* (40,000/t).*
The power directly as a function of time, for a 40 km race.)

d. Use the composition of functions from part c to find the power that Lance supplied to his bicycle.
(Answer: P(4440 sec) = 220 watts)

7. You will need a bicycle, measuring tape, and marker in order to do some counting and measuring.

a. What is the diameter of the rear tire of your bicycle?
Now look at the gears currently connected by the chain.
Count the number of teeth in the gear at the pedal (*teeth in the pedal gear*).
Count the number of teeth in the rear wheel gear (*teeth in the wheel gear*).
(You may want to mark a tooth with felt-tip, lift the wheel and move the pedal to count the teeth.)
(*Answers vary, but for example: diameter = 25.5 ".*
Teeth in pedal gear = 32
Teeth in rear wheel gear = 24)

b. The (**gear**) ratio, *teeth in the pedal gear* to *teeth in the wheel gear* ,
is equal to
the (**turn**) ratio, *turns of the rear wheel* to *turns of the pedal*.

For this exercise, **do not reduce your ratios.**
What is your **gear** ratio, *teeth in the pedal gear* to *teeth in the wheel gear*?
___/___
State the resulting **turn** ratio, *turns of the rear wheel* to *turns of the pedal*.
___/___
*(Answer: For our example, our **Gear** ratio is 32/24. Our **Turn** ratio is 32/24.)*

c. To help you think about why the gear and turn ratios must be equal, do the following using the numbers from the (unreduced) gear and turn ratios:
Multiply the number of *turns of the rear wheel* by the number of *teeth in the wheel gear*. What does this number mean?
Multiply the number of *turns of the pedal* by the number of *teeth in the pedal gear*. What does this number mean?
Do your numbers agree? Why?

(Answer: The total number of teeth moved in the rear wheel gear.
The total number of teeth moved in the pedal gear.
Yes. The chain pulls one rear wheel tooth for every one pedal tooth.)

d. Write a proportion using the number of turns of the rear wheel (*r*), the number of turns of the pedals (*p*), and your gear ratio.
(Answer: For our gear ratio, r/p = 32/24.)

Solve the proportion to write a formula for the number of turns of the rear wheel (*r*) in terms of the number of turns of the pedal (*p*).
*(Answer: For our gear ratio, 24 * r = 32 * p ⇒ r = (32/24) * p)*

Now write a formula for *p* as a function of *r*.
*(Answer: For our gear ratio, p = (24/32) * r.)*

Congratulations! You found the Inverse Function!

e. Write a formula for the distance traveled (s) by your bicycle as a function of the number of the turns of the rear wheel (*r*). (*Hint:* How far does your bicycle travel in one full turn of the rear wheel?)

Teacher Note: Students will probably use "d" for diameter, so distance has been denoted as "s".
*(Answer: For our 25.5 inch wheel, s = C * r = (π * d) * r = (π * 25.5) * r.)*

Use composition of functions to write a formula for distance (*s*) traveled as a function of the number of turns of the pedal (*p*).
(Answer: For our 25.5 inch wheel and our 32/24 gear ratio,
*s = (π * 25.5) * r = (π * 25.5) * [(32/24) * p].)*

f. Calculate the speed of your bike in miles per hour (mi/hr) using the selected gear and a pedal rotation of 60 revolutions per minute (rpm). Show your method.
(Answer: In our example, wheel = 25.5 inches and gear ratio is 32/24.
*Since d = r * t, r = d / t.*
*In our notation, rate = s / t = (π * 25.5 * (32/24) * p) / t*
*= (π * 25.5 * (32/24) * 60 turns) / 1 min*
≈ 6409 "/min ≈ 6 mi/hr.)

Test your method with the following data: If the distance traveled by one turn of the rear wheel is 85", and the gear ratio (pedal teeth to rear wheel teeth) is 52/14, and the cyclist is pedaling at 100 rpm, your method should yield a speed of about 30 mi/hr.

(Teacher Note: Your students can verify that their method is correct by succeeding with this hypothetical calculation.)

g. Under what road and weather conditions do riders use lower gears? Try to suggest three different conditions.
(Answer: Hills; gravel or other rough surface; an opportunity to rest)

Check your completed exercises with your teacher before continuing. Make certain your answers are correct because you may use some of the formulas or explanations in your presentation.